UNIT E
POPULATION, CLIMATE, AND MATHEMATICAL MODELING

DETAILED CONTENTS

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UNIT E

POPULATION, CLIMATE, AND ENVIRONMENT

“The weight of our civilization has become so great, it now ranks as a
global force and a significant wild card in the human future along with
the Ice Ages and other vicissitudes of a volatile and changeable
planetary system”

--Dianne Dumanoski

0 OBJECTIVES

1. To understand the factors that affect how populations of living organisms change with time
2. To understand steady growth, doubling time and growth rates
3. To understand the origins of steady growth in populations of living organisms
4. To learn to model population dynamics using computer spreadsheets.
5. To develop skills in using spreadsheet modeling to explore the behavior of dynamical systems
6. To understand the fundamental physical principles that drive the greenhouse effect
7. To develop a model of energy balance in the environment that can be used to explain how
increased atmospheric CO2 might cause global warming via the greenhouse effect
8. To learn more about the population dynamics, global climate change, and the process of
scientific research by undertaking an independent investigation.
0.1 Overview

Why Study Environmental Issues?

Nearly every day, on the news, there is at least one story that focuses on environmental issues. Some of these stories tell us that the sky is falling, while others tell us that everything is going to be just fine. What should we believe? How can we determine who’s telling the truth? How can we determine which issues are the most important? It all seems so confusing that many people assume that the average person can’t possibly understand enough of the science to make rational decisions about policies regarding the environment.

Nevertheless, we make these kinds of decisions when we vote, when we buy something, when we throw something away, and even when we decide to have children. While it is obviously unreasonable to expect that everyone should learn all the details of the science underlying our understanding of environmental issues, it is possible for us all to develop a basic understanding of our environment. With this basic framework, we can make rational decisions about environmental policy and broaden our understanding as it becomes necessary.

The primary goal of this unit is to give you the opportunity to learn some of the fundamental principles that form the foundation for our understanding of current environmental issues. This unit will focus on the ideas of growth, competing rates, modeling, and dynamic equilibrium. You will develop tools for understanding how scientists develop their knowledge of the environment, and you will explore how a simple model of growth can be used to explain a wide range of environmental (and physical) phenomena.

But before you begin, it will be valuable to think carefully about what environmental issues you think are most important. In the next activity, you will discuss your views on the most important environmental considerations facing society today.

Activity 0.1.1 Environmental Issues

a) Note: Compose your answer to this question before discussing the question with your group mates. Based on your knowledge and opinion, briefly describe what you think are the most important environmental issues facing the world today. Briefly describe why you think these issues are most important.
b) Now, discuss your responses with your group members and write down any new ideas mentioned by your group. Note: Your instructor may initiate a group discussion. If so, write down any new ideas brought up in this discussion.

c) Did anything you heard or learned in these discussion influence how you would respond to part a). If so, explain what you learned and how you would change your response.

Throughout this unit, you will explore some of these important environmental issues. Keep in mind that much of what you learn in this unit may be useful in developing your understanding of the other issues you discussed in the last activity.
The unit will begin by exploring how populations of organisms grow. In Section Error! Reference source not found., you will develop models to explore how various factors affect the growth of populations. In Section Error! Reference source not found., you will explore the physical principles that drive the greenhouse effect. Finally, in Section Error! Reference source not found., you will use the modeling skills you developed earlier in the unit in conjunction with your understanding of the greenhouse effect to develop a simple model for how an increase in atmospheric CO₂ could cause temperatures at the surface of the earth to rise.
In the last section you identified a number of global environmental issues that you felt were most important. It would be surprising if population growth/over-population was not on your list somewhere. As the Earth’s population grows, so do all of the affects that humans have on our environment.

In order to understand how population growth affects the global environment, we will take some time to explore the fundamental factors that affect the growth of populations of living organisms. In this section, you will begin to elicit some of the important factors that affect growth, and you will explore two methods for how we can define growth quantitatively.

You may notice that throughout this section we look exclusively at how populations of microscopic organisms grow. We focus on these smallest of living organisms for two reasons. First, since these organisms reproduce rapidly and take up very little space, it is possible for us to observe population growth over a matter of hours or days rather than years. Secondly, since these organisms are small, it is very easy to control the environment in which they grow. Thus, we can observe the fundamental behavior of growing populations without the complications introduced by a changing environmental conditions. While these observations do not exhibit all the peculiarities of real world population growth, they give us valuable insight into the mechanisms of population growth.

You will need some of the following equipment for the activities in this section:

- Time lapse video of the growth of a bacteria colony
- Videopoint (optional video analysis software)
- Computer spreadsheet program (Files available for Microsoft Excel)

1.1 The Factors that Affect Growth

Throughout the rest of this unit, we will talk about the growth of organisms, but what do we mean by growth? In the biological sense, growth can mean a variety of things. For the most part, however, we think of growth as either how much a single organism grows (it’s change in height or weight), or as the change in the number of individuals in a population of organisms. While the growth of individual organisms is very interesting in its own right, population growth is more relevant for our purposes in this unit. Thus, for the rest of this unit, unless otherwise specified, when we say growth, we are talking about population growth.

As we begin to investigate how populations of living organisms grow, it is natural to ask the question, “What factors affect the growth of living organisms?” In the next few activities, you will explore this question in more detail, by looking at the growth of some populations of microscopic organisms.

In the next activity, you will observe the growth of Escherichia coli (E coli) bacteria. E. coli are very common bacteria that live in the lower intestines of many mammals including humans. You may have heard of E. coli before. On occasion they make the news, when a stream or lake becomes contaminated with them. While E. coli are harmless in your lower intestines, they can cause violent illness if ingested.

Luckily for us, we won’t be handling them. Rather, we will be watching time lapse movies of their growth. The movie you will observe in the next activity was made by placing E. coli bacteria on a plate of agar. The colony of bacteria was then placed under a microscope and maintained at a constant temperature. Approximately twice each
minute, an image of these bacteria was captured. These images were then compiled into the movie you will watch.

![Bacterial Images]

**Figure E-2:** Screen shots from a time lapse movie of Escherichia coli (E.coli) undergoing division.

### Activity 1.1.1 Watching Bacteria Grow

a) Watch the video, E010101.mov. Describe what you see below.

b) Briefly describe how the colony of bacteria changes over the course of the movie. How did the colony of bacteria start? How did it change over the course of the movie? And, what do you think will eventually happen to the colony of bacteria?
c) Within your groups, brainstorm and write down all the factors that you think might influence the total number of living E. coli bacteria in the colony at any one time. Write your list below.

Figure E-3: An example of a hierarchical diagram of the factors that affect a bank account. The factors that directly affect the bank account balance are the amount of money deposited and withdrawn from the bank account each month. The amount withdrawn from your account depends on your bills, purchases and taxes. These factors are then influenced by your spending habits and the particular tax laws in your area.

d) Now, think about how you might group these factors into more general categories. That is, identify which of these factors influence other factors. Organize these factors hierarchically based which ones most directly influence the number of living E. coli bacteria, and then which one influence those factors. Figure E-3 shows an example of a hierarchical diagram for a bank account.
What Factors Affect the Growth of Humans on Earth?

In the last activity, you identified and organized some of the factors that affect the growth of the population of a colony of E. coli bacteria. The factors that influence the population of bacteria colonies also affect animal populations. In fact, it is likely that many of the factors you identified are related to the fundamental factors that affect the growth of any population of organisms, including the human population. In the next activity, you will identify and organize some of the factors that affect the size of the human population on Earth.

Activity 1.1.2 The Human Population

a) Within your groups, brainstorm and list all the factors you can think of that affect the number of humans present on earth.

b) Now, organize these factors into a hierarchical diagram like you did in Activity 1.1.1 d).
c) Which factors most directly influence the human population?

Your instructor may lead you to discuss the ideas that you came up with in the last two activities.

**Checkpoint Discussion: Before proceeding, discuss your ideas with your instructor!**

### 1.2 HOW DO POPULATIONS GROW?

In the last two activities you began to speculate about what factors affect population growth. While this is a good start for our investigation, it does not afford us very many options for quantitative study. In order to develop a more quantitative approach to understanding population growth we must first make some close observations of how populations grow, and then use what we observe to develop a quantitative definition of growth.

**How do E. coli Grow?**

In the next activity, you will look more closely at how a colony of E. coli grow over time. Note: If you have access to video analysis software like Videopoint, you can open the file E010201.vpt and use the count function of Videopoint to count the number of bacteria in each frame of the video. If you do not have access to Videopoint, open the file E010201.mov using the free version of Quicktime and count the number of bacteria in each frame.
Activity 1.2.1 How does an E. Coli Colony Grow?

a) Play the video E010201.mov and describe how the population of E. coli bacteria is growing as a function of time.

b) What quantities might you use to describe the condition of the E. coli population.

c) Using either the file E010201.vpt or E010201.mov count the number of e coli bacteria in each frame of the video and record your counts in the table below and graph your results.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Time (Minutes)</th>
<th># of Bacteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>660</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>1320</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>2640</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>3300</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>3960</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>4620</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>5280</td>
<td>88</td>
<td></td>
</tr>
</tbody>
</table>
d) In your own words, describe how the population of bacteria grew in this movie.

e) Within your group, come up with a clear, quantitative definition for what you mean by growth and write it below.
f) Using your definition of growth, determine how much the population grows in the first 11 minutes. Use your definition to determine the amount by which the population grows during each 11 minute period and record your results below.

<table>
<thead>
<tr>
<th>Time (Minutes)</th>
<th>Growth of Bacteria Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 11</td>
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<tr>
<td>11 – 22</td>
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<tr>
<td>22 – 33</td>
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<td>33 – 44</td>
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<td>44 – 55</td>
<td></td>
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<tr>
<td>55 – 66</td>
<td></td>
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<tr>
<td>66 – 77</td>
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<tr>
<td>77 – 88</td>
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</tbody>
</table>

g) As time goes on does the speed at which the population is growing change. That is, what happens to the rate of growth of the population as time increases.

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*Defining Growth*

In the last activity, you took a stab at defining growth. In the next couple of activities, we will look more closely at some of the peculiar properties of population growth under controlled conditions. These observations may help us come up with some more sophisticated ways of defining growth.

In the next activity, you explore one method of quantifying the amount of growth a population undergoes.
Activity 1.2.2 Doubling Time

a) Using the tables and graph from Activity 1.2.1, estimate the amount of time it took for the number of E. coli bacteria to double.

b) Next, determine the amount of time it took for the number of E. coli to double a second time and fill in the table below.

<table>
<thead>
<tr>
<th>Doubling</th>
<th># of Bacteria</th>
<th>Doubling Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
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<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
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</tbody>
</table>

c) Do you notice any relationship between the doubling times in the chart above? If so, describe this relationship.

d) Estimate how long you think it will take for the number of bacteria in the colony to double a 5<sup>th</sup> time.
e) Do you think the population of bacteria will continue to double at this rate forever? If so, describe the consequences of this kind of growth and explain why the world is not filled with E. coli bacteria. If not, describe what you think might eventually limit the growth of the E. coli population.

Checkpoint Discussion: Before proceeding, discuss your ideas with your instructor!

In the last few activities, you looked at how a population of E. coli bacteria grew over time. You measured the doubling times for the population and likely found that the doubling time was fairly constant. The fact that the doubling time for the colony of E. coli is nearly constant is quite curious. Off hand, there doesn’t seem to be any obvious reason why this should be so. Nevertheless, it seems to suggest a pattern, thus, as scientists we should take note.

One of the primary activities that scientists engage in is looking for patterns. While it is often the case that these patterns turn out to be entirely coincidental, sometimes, the patterns we observe give us clues about the underlying principles that govern the system we are investigating. For the time being, we will take note of the fact that the doubling time in E. coli seems to be constant. In the future, we may be able to combine this information with other observations to further develop our understanding of how populations grow.

**Fractional Growth Rate**

In the next activity, you will look at another way to measure the growth of a population. In this case, you will look at the population growth of microscopic marine organisms called phytoplankton. Like E. coli, phytoplankton are microscopic organisms. It is often easiest to do population studies with microscopic organisms, because they grow quickly enough that it is possible to observe the growth of the population through many
generations in a short period of time and because it is reasonably easy to control the environments in which they grow.

In the next activity, you will examine data depicting the growth of a population of Isochrysis (phytoplankton) over the course of 9 days.

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**Activity 1.2.3 Measuring the Growth Rate**

a) Using the graphs below, determine the size of the population at the end of each day. Use this data to fill in the table in the spreadsheet E010203.xls.

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**Figure E-4**: Isochrysis phytoplankton. Isochrysis are rapidly growing microscopic single-celled organisms that live in the ocean. The Isochrysis shown above are about 5µm long.
b) Devise a method for determining the fractional increase (% growth) the population undergoes each day. Discuss your method with your group mates, come to a consensus about how you should calculate this, and describe your method below.

c) Now, use your method to calculate the percentage growth or growth rate of the population of Isochrysis for each day. Enter your results in the spreadsheet. **Note:** When you have completed this entire activity, print out your spreadsheet and include it in your notebook.
d) Describe any patterns you observe in your calculated growth rates?

e) Use the graphs above to determine the doubling times as before.
Enter your data in the table below.

<table>
<thead>
<tr>
<th>Doubling</th>
<th># of Isochrysis</th>
<th>Doubling Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>800</td>
<td>NA</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
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</tbody>
</table>

f) Describe any patterns you observe in the doubling times you tabulated above.

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Checkpoint Discussion: Before proceeding, discuss your ideas with your instructor!
Your instructor may initiate a group discussion about your observations and analysis in the last few activities.

You may have noticed in the last activity that the growth rate for Isochrysis was nearly constant over the course of the 9 days during which it was observed. As before, this pattern is quite curious. You may have also noticed that for much of the data, the doubling time was nearly constant. Do you think that these two patterns might be related.

In the next section, we will try to use what we know about how organisms grow to develop some models for population growth. By comparing these models to some observations, we may be able to develop a better understanding of population growth and gain some understanding of the observations we have made in this section.
In this section, you will develop some simple models of how populations grow. You will develop rules for your model, apply these rules and see how the population grows as a function of time.

For simplicity’s sake, we will assume that we are talking about a population of humans. That said, the models we will generate throughout this unit reflect generalizable if somewhat simplified principles regarding population growth.

As we begin to develop models for population growth, it will be helpful to look back at what we discussed on the first day of the unit (See Activity 1.1.2). Based on your hierarchical diagrams for the factors that affect human population growth, you may have determined that the two factors that most directly influence population are the number of births and the number of deaths at any given time. With this in mind, we will begin to build some simple models taking into account only these two factors.

You may need some of the following equipment for the activities in this section:

- Beans and cups
- Computer spreadsheet program. (files available for Microsoft Excel)

**Modeling in Science**

Among the many activities that scientists engage in, modeling is one of the most important. But what do we mean by modeling in this context. As is often the case, we are using the term modeling in a very specific sense. When *modeling*, scientists develop a physical representation, a system of rules, a system of equations, etc. that represents some of the relevant features of the physical system they are interested in investigating. We call this simplified representation a *model*. 
Typically, scientists start out with a very simple model even though it only roughly mimics the properties of the system they are interested in. They then observe how the system behaves when they vary the parameters (input variables) of the model. Once they have completely explored the simple model, they gradually increase the complexity of the model so that it more closely reflects the behavior of the situation. Step by step, they add layers of complexity to the model and at each stage, they explore in detail how the new model behaves. Through this process, scientists can develop an understandable model that more closely mimics the behavior of the real system.

2.1 Modeling Populations with Beans

Arguably, the simplest model we could generate for population growth is one in which the population grows by a set amount each generation. For example a population might grow by 10 people each year. While this extremely simple model will serve as our starting point for modeling population growth you may find that it is not a very good model.

To help make this modeling more concrete, you will begin by using a pile of beans to represent a population of people. Thus, if you have one bean in your pile, your population is one person. If you have 200 beans your population is 200 people. If a person is born, you will represent that by adding one bean to your pile. If a person dies, you will represent that by removing one bean from your pile.

Activity 2.1.1 Modeling Populations with Beans: Part I

a) Begin by observing what happens to your population if you use the “simplest” model described above. Begin with 10 beans. Assume that two people are born each generation. Fill in the table below using your beans to help you keep track of what is going on.

<table>
<thead>
<tr>
<th>Generation</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>10</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
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<td>6&lt;sup&gt;th&lt;/sup&gt;</td>
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<td>7&lt;sup&gt;th&lt;/sup&gt;</td>
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<td>8&lt;sup&gt;th&lt;/sup&gt;</td>
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<td>9&lt;sup&gt;th&lt;/sup&gt;</td>
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<tr>
<td>10&lt;sup&gt;th&lt;/sup&gt;</td>
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</tbody>
</table>
b) Does this model do a good job of reflecting how populations grow? What might be missing from this model?

c) In the last model, we only took into account the number of people born each generation. Maybe our model could be improved by taking into account deaths as well. Assuming that 2 people are born each generation and one person dies each generation, show what will happen to the population over the course of 10 generations in the table below.

<table>
<thead>
<tr>
<th>Generation</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>10</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td></td>
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<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
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<td>10&lt;sup&gt;th&lt;/sup&gt;</td>
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</tbody>
</table>

d) Graph the data from parts a) and c) on the following figure and compare them to the growth curves you looked at in Activity 1.2.1 and Activity 1.2.3. How are they different? How are they similar?
e) Based on your intuition and experience with how real populations grow, describe what you think should happen to the population after a long time (approximately 1000 generations). What do you think will happen to the number of people being born each generation after 1000 generations? What do you think will happen to the number of people dying after 1000 generations?

f) Based on the model in part c) (in which two people are born and one person dies each generation), what will happen to the population far in the future, after 1000 generations? What will happen to the number of people being born each generation after 1000 generations? What will happen to the number of people who die each generation after 1000 generations?
g) Compare your answers in parts e) and f). In what ways do the simple models you used in this activity reflect how populations really grow? In what ways do they seem to fall short? Explain the reasoning behind your answers.

While the models we used in the last activity were certainly simple, the rules we chose were somewhat arbitrary. There were some things about the models that did reflect how real populations grow (the population increased with time, the rate of increase was smaller when we incorporated a death rate, etc…), but there were also some things that did not reflect how real populations grow (the shape of the population curve as a function of time was very different than what we observed in Section 1).

Building Models Based on Knowledge

While the two models developed so far in this section are a good start, they have some major drawbacks. In particular, they do not take into account our knowledge about how organisms grow and reproduce. In the next activity, you will attempt to build some population models with rules based more closely on what you know about how organisms reproduce.

Activity 2.1.2 Revising Your Model

a) If you have a population of 10 people and 2 people are born each generation, how many people do you think would be born each generation if you had a population of 20 people? How many people do you think would be born each generation if you had a population of 100 people? 5000 people? Carefully explain your reasoning? Note: You may find it useful to use a specific example to work out your answers to this question.

b) Based on your responses to part a), discuss in your groups how you might improve the models from the last activity to incorporate what you know about how people reproduce. Describe your ideas below.
c) Devise a simple model that incorporates these ideas. List a simple set of rules below that dictate how the population will change with each generation.

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**Checkpoint Discussion: Before proceeding, discuss your ideas with your instructor!**

Your instructor may lead a group discussion about your responses to the last activity. Be prepared to explain your model to the class and describe the reasoning that went into your choice of model.

**Activity 2.1.3 Modeling Populations with Beans: Part II**

a) Using the revised model that the class settled on in the previous discussion calculate the population for the first 8 generations. Begin with a population of 10 people and assume that 2 people are born the first year. As before, use beans to represent people, keep
track of your population in the table below, and graph the population below.

<table>
<thead>
<tr>
<th>Generation</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>10</td>
</tr>
<tr>
<td>1\textsuperscript{st}</td>
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<td>2\textsuperscript{nd}</td>
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<td>3\textsuperscript{rd}</td>
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<td>4\textsuperscript{th}</td>
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<tr>
<td>5\textsuperscript{th}</td>
<td></td>
</tr>
<tr>
<td>6\textsuperscript{th}</td>
<td></td>
</tr>
<tr>
<td>7\textsuperscript{th}</td>
<td></td>
</tr>
<tr>
<td>8\textsuperscript{th}</td>
<td></td>
</tr>
</tbody>
</table>

b) Compare your data to the growth curves you looked at in Activity 2.1.1. How are they different? How are they similar?
c) Compare your data to the growth curves you looked at in Activity 1.2.1 and Activity 1.2.3. How are they different? How are they similar?

2.2 Modeling with Spreadsheets

In the last section, you carried your model forward in time by doing hand calculations, or by counting beans. While this method is perfectly valid, it is quite time consuming. In addition, the time required to do these calculations by hand will go up dramatically when we begin to look at more sophisticated models and/or try to propagate our models for more than a few generations.

Luckily, you have a fantastic tool for this kind of modeling right at your fingertips. As you saw in Section 1 spreadsheet software can dramatically speed up your calculations. In the next activity, you will use spreadsheet software to regenerate the models you explored in the first part of this section.

Activity 2.2.1 Building Spreadsheet Models

a) Open up the spreadsheet E020201.xls. This spreadsheet contains data for the first model you looked at in this section. Modify this spreadsheet to include all three of the models you have looked at in this section and make a graph that shows all three populations as a function of time. Print out your graph and include it in your spreadsheet. Note: You think carefully about how you want to organize your spreadsheet. It is always a good idea to label things clearly. Also, you may find it helpful to follow the pattern used for the first model as you set up calculations for the other models. Finally, you may find it useful to save your spreadsheet files for later use.

b) Extend your models to 100 generations. What are the populations for each model after 100 generations? Print out a graph of the
population over 100 generations and include it in your activity guide.

c) Look closely at the data and find the doubling times for each model. Tabulate these below.

<table>
<thead>
<tr>
<th>Doubling Time</th>
<th>Model 1: Births/Gen=2</th>
<th>Model 2: Births/Gen=2 Deaths/Gen=1</th>
<th>Model 3: Births/Person/Gen=0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4th</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>5th</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6th</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d) Describe any patterns you notice in the doubling time for each of the models.

e) Determine the growth rate (% growth) each generation for each model. Calculate the growth rate using the spreadsheet, plot the growth rates as a function of time (generation). Plot the growth rates for all three models on the same graph, print out this graph and include it in your activity guide.
f) Describe any patterns you notice in the growth rate for each of the models.
Two Types of Constant Growth

In the last few activities, you’ve explored some very simple models for population growth. If you were to try to name these models, what would you call them? Your first impulse might be to call the first model a constant growth model since the population grows by a constant amount each generation. You might also consider calling the third model a constant growth model because the population grows by the same fraction of the population each generation.

In fact, scientists have already given names to these two types of “growth”. Scientists call the type of growth you saw in the first model, linear, because a graph of the population as a function of time is a straight line. Linear growth is characterized by a constant increase in the population each year. Both of the first two models you investigated in Activity 2.1.1 are linear, since they both produce a straight line graph of population as a function of time. The third model you developed has many names. It is sometimes called a constant growth model because the population increases by the same fraction each generation. It is also called an exponential growth model because the population vs. time curve is the same shape as an exponential curve.

In the next activity, you will explore some of the important properties of linear and exponential population growth models. Use your saved graphs from Activity 2.2.1 for the following activity. Note: Remember to save your work from these activities on separate worksheets and save them for use in later activities.

Activity 2.2.2 Linear Models

a) Look at the first model (the linear model with only a birth rate). If you increase the birth rate, how does it change the graph of population vs. time? If you decrease the birth rate, how does it change the graph?

b) Now, look at the second model (linear model with birth rate and death rate). If you increase the death rate, how does it change the graph of the population vs. time? If you decrease the death rate, how does it affect the graph?
c) Assume that a boat carrying 80 people wrecked off the coast of a deserted island. Assume all 80 people made it safely to shore, settled in and began to reproduce. Use your spreadsheet model to model this population. Assume that approximately 2 people are born each year and that one person dies every 2 years. Graph the population and insert it in your activity guide.

d) If after 100 years, a ship happened to stop by the island. How many people would they find? How many people would they find after 1000 years.

e) Determine the annual growth rate (people per year) for the island population?

f) Assume that after 50 years on the island, half the population was killed in an epidemic. Model this population using your spreadsheets. Graph the population as a function of time for 100 years after they were stranded. Print out your graph and include it in your activity guide.
g) If after 100 years, a ship visited the island, how many people would they find.

While these linear models provide a simple introduction to spreadsheet modeling, they do not do a very good job of reflecting how real populations grow. However, the exponential model showed some promise of behaving more like a real population.

**Activity 2.2.3 Exponential Models**

a) Look at your third spreadsheet model (exponential or constant fractional growth rate). What happens to the population vs. time graph when you increase the birth rate (births per person per generation)? What happens when you decrease the birth rate?

b) Now, modify this model to incorporate a death rate (deaths per person per generation). What happens to the population vs. time graph when you increase the death rate? What happens when you decrease the death rate? What happens to the population vs. time graph if the death rate is larger than the birth rate? What happens to the graph if the birth rate is higher than the death rate? What happens to the graph if the birth rate is equal to the death rate?

c) Set your birth rate to 2 births per person per generation and your death rate to 1 death per person per generation. Determine the % population growth per generation for this model.
d) Fill in the table below using the birth rates and death rates shown.

<table>
<thead>
<tr>
<th>Birth rate (births per person per generation)</th>
<th>Death rate (deaths per person per generation)</th>
<th>% Growth Rate (% increase in pop. per generation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>697</td>
<td>696</td>
<td></td>
</tr>
</tbody>
</table>

e) What pattern do you notice in the preceding table? How is the % growth rate related to the birth rate and death rate in this model? Devise an equation relating birth rate, death rate and % growth rate.

f) Assume that a boat carrying 80 people wrecked off the coast of a deserted island. Assume all 80 people made it safely to shore settled in and began to reproduce. Use your exponential spreadsheet model to model this population. Assume that two people are born the first year and that the % growth rate of the population remained constant. Also assume that one person died during the first 2 years and that the % death rate remained constant. Graph the population and insert it in your activity guide.
g) If after 100 years, a ship happened to stop by the island, how many people would they find? How many people would they find after 1000 years.

h) Determine the annual growth rate (people per year) for the island population? Determine the annual % growth rate (% of the population per year) for the island population.

i) Assume that after 50 years on the island, half the population was killed in an epidemic. Model this situation using your spreadsheets. Graph the population as a function of time for 100 years after they were stranded. Print out your graph and include it in your activity guide.

j) If after 100 years, a ship visited the island, how many people would they find.
k) Compare your responses to Activity 2.2.3 part j) and Activity 2.2.2 part g). Describe how the results of the linear and exponential models differ.

Checkpoint Discussion: Before proceeding, discuss your ideas with your instructor!

In the last two activities, you used spreadsheet modeling to predict how a real population would grow over time.

Modeling a Real Population.

In the next activity, you will use the exponential growth model to make some predictions about how the human population should have changed over the past 4000 years. Then you will compare your predictions to how the population has actually changed.

In the next activity you will build a model for how the human population has grown since the year -2000 (2000 B.C.). Before you construct your model, you must first come up with some reasonable estimates for the parameters that will affect the population. In the previous models, there have been four parameters that you have used to control your population models. These include birth rate (BR), death rate (DR), initial population (PI), and the length of time for a generation (GL).

You have already explored birth rate and death rate fairly extensively, but it is worth reiterating that these rates are a measure of how births (or deaths) occur per person each generation. Thus, a birth rate of 1 implies that there was one birth for each person during each generation, or that each couple had 2 children. Similarly, a death rate of 1 implies that every person alive at the end of the last generation will die during this generation. It seems to make sense that the death rate should always be 1, since everyone in any generation will eventually die. That said, we can use the death rate as an indirect adjustment of lifespan. You will have to choose a birth rate and a death rate for your model in the next activity.

The initial population is somewhat self explanatory, but will have an important affect on the future growth of the population. In the next activity, we will use a historical estimate of the human population (~27 million people) in the year -2000 for our initial population.

The last parameter, generation length seems obvious enough, but is a somewhat arbitrary parameter that will have more of an affect on our model they one might think at first. The important thing to keep in mind about the generation length is that it is intimately connected to the birth rate and death rate, since BR (DR) is measured in number of births (deaths) per person per generation. Thus, if we change the generation length, we must take into account how this will affect the birth and death rates. For the next activity, we will assume for convenience that the generation length (GL) is 25 years.
a) Create a new sheet in the population modeling spreadsheet you started in Activity 2.2.1. Construct a spreadsheet using the exponential growth model and a 25 year time between generations to model the human population from -2000 to 2000. Note: Remember to incorporate the birth rate and death rate as variable parameters in your spreadsheet.

b) Discuss with your group what you think would be reasonable values for the birth rate and death rate. Record your predicted values for birth rate and death rate below and indicate the reasoning behind your choice of these rates.

c) Use your spreadsheet to model the population of the earth from -2000 to 2000. Graph your results, print them and include them in your workbook.

d) Open up the file E020204.xls and copy the world population data into your model spreadsheet. Graph your model and this data together on the same plot. Print this graph and include it in your spreadsheet. Compare your model to the real population data for the Earth. How are they similar, how are they different.
e) Adjust the birth rate and death rate in your model so that your model more accurately reflects the real data. How did your birth and death rates change? What does this tell you about the true birth and death rates.

f) After you have adjusted your model to get the best fit you can, print out a graph of the real data and your model and include it here. How is your model similar to the real data? How is it different?

g) Based on your comparison of your model with the real data, what can you conclude about your model? How might you improve your model so that it more accurately reflects the real data?

h) Look carefully at the graph of the real data for the human population on earth over the last 4000 years. Does the data seem to fit a linear model, an exponential model, something else? How might you determine if the human population has experienced exponential growth over the past 4000 years.
Clearly, our simple model doesn’t seem to fit the growth of the human population very well. Even so, it does mimic some of the important behaviors of the population and is simple enough for us to understand well. It’s not too surprising that our model isn’t perfect, since we have only included a few of the factors you identified in Activity 1.1.2. In the next subsection, we will begin to incorporate some of these into our model, but for the time being, we will look more closely at how our model fails.

Activity 2.2.5 Why Doesn’t Earth’s Population Grow Exponentially

a) Discuss with your groups, why you think the Earth’s population hasn’t grown exponentially over the last 4000 years. Record your ideas below.

b) Which of the factors that influence the growth of a population might have changed over the course of the last 4000 years, resulting in the variations from exponential growth that you observed in the last activity? What events in human history might events might have caused these changes?
Exponential Growth and Constant % Growth

Earlier in this unit you likely discovered that the % growth rate was constant for our exponential growth model. This is true of any exponentially varying system. In fact, we can define exponential growth with this fact. An exponentially growing value is one in which the value experiences the same % growth over any equal time interval. Thus, if the population of a small town is growing exponentially and it increases from 1000 to 1500 between 1950 and 1960. If it continues to grow exponentially, it will grow by the same percentage (50%) in any 10 year interval. Thus, its population should grow from 1500 to 2250 between 1960 and 1970 or from 2000 to 3000 between 1967 and 1977. Note: Recall that % growth and fractional growth refer to the same quantity.

You will use this fact about the % growth rate and exponentially growing systems to help you get a better understanding of how your exponential model is different from the real human population on Earth.

**Activity 2.2.6 Growth Rates in a Real Population**

c) Calculate the % growth for each interval of 1000 years in the given world population data and record it in the table below.

<table>
<thead>
<tr>
<th>Millenium</th>
<th>Average Annual % Growth Rate (% increase in pop. per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2000 to -1000</td>
<td></td>
</tr>
<tr>
<td>-1000 to 1</td>
<td></td>
</tr>
<tr>
<td>1 to 1000</td>
<td></td>
</tr>
<tr>
<td>1000 to 2000</td>
<td></td>
</tr>
</tbody>
</table>

d) In your spreadsheet, calculate the % growth of the population for each time interval included in the spreadsheet. Calculate the % growth for both the data and your model. Plot the % growth as a function of time for your model and the real data on the same graph.

e) Compare the % growth rate for your model with the % growth rate for the real data. How are they similar? How are they different?
f) What do you observe about the growth rate for the human population on Earth over the past 4000 years? Has it been constant? Has it changed? If it has changed, when has it changed most dramatically?

g) Do you think that these changes in growth rate correspond to any important events in human history? If so, which changes do you think correspond to which events? **Hint:** You may be able to identify population events associated with the Plague during the middle ages, the World Wars, and the baby boom in the 60’s

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**Checkpoint Discussion:** Before proceeding, discuss your ideas with your instructor!

2.3 Incorporating Limitations to Population Growth

In Section 2.2 we developed some simple models for how populations grow. We found that exponential population models are better than linear models for predicting how populations will grow, but that these models have some significant limitations. This is not too surprising since our model does not take into account any of the secondary factors you identified in Activity 1.1.2. It is reasonable to assume that incorporating some of these factors into our model might help our model give more realistic predictions of how a real population might grow. In this section we will incorporate some of these factors into our model and explore how they influence population growth in the context of our model.

**Resources as a Limit to Population Growth**

In Activity 1.1.2, you identified many factors that affect population growth by influencing the birth and death rates. Some of these factors might include: availability of food, availability of energy, availability of water, availability of shelter, human waste, toxic waste, polluted air, polluted water, human excrement, trash etc. It is possible to divide all these factors into roughly two categories: available resources and pollution.
Obviously, we expect that with more available resources (more food, water, energy, etc.) the death rate due to lack of these resources will fall. It is also reasonable to conclude that with an increase in pollution (human waste, toxic waste, air pollution, water pollution, etc.) the death rate will rise. Thus, we have two obvious factors that will affect population growth. In addition, these factors will be affected by the population since with more people, more resources will be consumed and more pollution will be created.

As before, we want to construct a simple model, Thus, we will begin by incorporating only one new factor into our growth model. This factor could just as well be a limitation on the amount of available resources, the build up of pollutants in the environment, or some other factor. For the sake of simplicity we will look at how a limitation on the amount of available food affects the population. Note: It is possible to treat any limited resource or pollutant in the same way we will treat food here. Later in this section we will work with a similar model but lump food, water, energy, etc. together into one general quantity called resources.

In the next activity, we will begin by looking at how the available food might be influenced by the population. As before, we will begin by using some simple rules and counting beans to help us keep track of how food will be used as the population grows.

### Activity 2.3.1 How do Populations Consume Resources?

a) Assume that two people are stranded on a deserted island with limited resources. In this activity, we will model how their population would grow over time. For our purposes, white beans will represent people and red beans will represent food. Begin with 2 people (white beans) and 50 units of food (red beans). Assume that each generation, your population will grow by 50% (birth rate = 100%, death rate = 50%) and that for each generation they are alive, 1 person uses up 1 unit of food (1 red bean). Thus, each generation, the available food will decrease by the number of people who were alive in the previous generation. Fill in the table below keeping track of how many people you have and how many units of food you have each generation.

<table>
<thead>
<tr>
<th>Generation</th>
<th>Population</th>
<th>Food</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>1st</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td></td>
<td></td>
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<td>3rd</td>
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<td>6th</td>
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</tr>
<tr>
<td>7th</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8th</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
b) How long does it take for the food to run out? What do you think will happen to the population on the island after all the food runs out.

c) Imagine that the people on the island found another food source so that the total amount of food available to them was 100 units instead of 50. Predict how much longer you think the food will last?

d) Build a spreadsheet for this model and begin with 50 units of food. How long does it take for all the food to run out? Increase the amount of available food to 100 units. How long does it take for all the food to run out? Compare your predictions in part c) with your observations from the model.
e) If the people on the island found a huge supply of food (~1,000,000 units) predict how much longer do you think the food would last?

f) Increase the amount of initial food in your model to 1,000,000 units in your spreadsheet. Compare your predictions from with what you observe.

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**Checkpoint Discussion: Before proceeding, discuss your ideas with your instructor!**

In the last activity you modeled the consumption of a limited supply of food by a population. You probably noticed that as the population grew, its food consumption grew. As a result, the amount of available food decreased and eventually went to zero. With no food left, it is only reasonable to assume that the population would all die off.

*Feedback*

This interaction between the population and the food supply (availability of food allows the population to grow, the population consumes food, lack of food causes the population to decrease) is known as a feedback because one element of the system affects another which in turn affects the first element. The results is a kind of causal loop in which a change in one factor (the population for example) affects that factor indirectly through another factor (food supply for example).

You probably noticed that eventually, both the population and the available food in your spreadsheet dropped below zero. Of course this does not reflect how a real population would behave. In a real population, when the available food dropped to zero, the whole population would die. In order to develop more accurate models for how populations grow and consume resources, we need to incorporate this simple fact.

In the next activity, you will build this simple feedback into your spreadsheet model.

**Note:** To do this, you will need to use the *if()* function in your spreadsheet. In Microsoft Excel, the command works like this.

```
=if(condition, then, else)
```

Where *condition* is an inequality like *A>B* or *C6<$D$7*, *then* is the value if the condition is true, and *else* is the value if the condition is false. If you have trouble using this command, use Excel help to search for the *if* function.
Activity 2.3.2 Building Feedback into a Spreadsheet Model Using the IF Function.

a) Discuss with your group how your model should behave when the amount of available food goes to zero. What should happen to the amount of food after that time? What should happen to the population?

b) Using the IF function, devise formulas for the population and available food that behave as you describe in part a). Write your formulas below and discuss them with your group or instructor if you are unsure of your formulas.

c) Enter these formulas into your spreadsheet. Modify your formulas until you get your model to behave appropriately, then graph your models, print them out and include them in your activity guide.

You may have noted that the model we developed in the last two activities is quite oversimplified. There are at least two important problems with this model. The first, is that, in our model, as we use up existing resources, no more are produced. The second problem is that our model population doesn’t seem to recognize that resources are being depleted until they are all gone. The first problem is fairly easy to deal with, so we will work with it first. The other problem is somewhat more sophisticated, so we will leave it until a bit later.
Figure E-6: Population model with feedback. Notice that the population grows to some value and then drops to zero. This occurs when all the resources have been used up. While this model has some realistic features, it is still not very realistic. In the next few activities, you will construct a much more realistic model of population growth limited by resources.

**Renewable and Non-Renewable Resources**

In the models we used in Activity 2.3.1 and Activity 2.3.2, whenever resources were used up by the population, they were gone forever and no other resources were produced to replace them. While this kind of model reflects the true properties of some resources like fossil fuels, it does a bad job for other resources like food and clean water.

Scientist refer to the first kind of resources as *non-renewable resources* because there is a limited supply and once that supply is used up, the resource is gone forever. A good example of non-renewable resources are fossil fuels. We have a large, but finite supply of fossil fuels available. Once these fuels are used up, there is no simple way of producing more.

We refer to the second kind of resources as *renewable resources* because some of the resource can be renewed. Food is a good example of this kind of resource. We consume food on a regular basis, but we also grow new food all the time. Clearly there are some
limits to how much food we can grow, but as long as we have a good environment in which to grow things, we will always be able to grow more food.

In the next activity, we will incorporate renewable resources into our model

Activity 2.3.3 Renewable Resources.

a) In Activity 2.3.1 and Activity 2.3.2, you developed spreadsheet models that kept track of non-renewable resources. How might you modify your model so that it keeps track of renewable resources only?

b) Graph a prediction of what you think your population will look like as a function of time with this new model. Note: Remember to label your axes. Explain the reasoning behind your prediction. Why do you think your graph will look this way?
c) Copy your spreadsheet from Activity 2.3.2 to a new worksheet and modify it so that your model takes into account renewable resources. **Note:** One easy way to do this is to fix the available resources to some value, for example, 1000 resource units per year. Graph the population as a function of time and include it in your activity guide.


d) Does your model agree with your prediction? Describe how your model is similar to and/or different from your prediction.

e) Why do you think your model did not fit your prediction. How might you improve your model to make it agree more with what you expect to happen?

You may have noticed in the last activity, that when the population exceeded the available resources, it either leveled off immediately, or dropped to zero (depending on how you set up your feedback in Activity 2.3.2. In essence, this is due to the second problem we identified at the end of Activity 2.3.2, i.e., that our model assumes that the population will grow until all available resources are gone. If this was how populations really consumed resources, we would certainly be in trouble, because the population would continue to grow until all the available resources were exhausted at which time the entire population would die. Luckily, this doesn’t seem to reflect how real populations respond to limited resources. In real systems, resource limitations begin to affect population growth at least some time before the resources are totally depleted. But how can we make our model reflect this behavior?
Before we try to solve this problem it may be helpful to review the models we have already developed and look at them in a variety of ways. In Activity 1.1.2 you produced a hierarchical diagram of factors that affect the human population on Earth. You organized this diagram based on how directly each factor affected the population.

A good example of the types of models you have been developing is a bank account. In the simplest bank account model, the bank account balance is only affected by amount of money deposited and the amount of money withdrawn from the account. This example is depicted in Figure E-7. This figure depicts a simple model in which the account balance is directly affected by only two things, deposits and withdrawals. In addition, the deposits is influenced by one other element, interest which is in turn influenced by the account balance.

**Figure E-7:** Hierarchical diagram for a simple model of a bank account. Notice that arrows show direct dependences. The Interest depends directly on the Account Balance, the Deposits depend directly on the Interest.

In addition to these hierarchical diagrams, you have been constructing spreadsheets to help you track the behavior of each of your models. In this type of spreadsheet model, you have been calculating new values for the population based on it’s previous values.

Following the bank account example in Figure E-7, you might come up with the following equation to relate the new bank account balance (AccBal\textsubscript{new}) with the old account balance (AccBal\textsubscript{old}).

\[
AccBal_{\text{new}} = AccBal_{\text{old}} + \text{Deposits} - \text{Withdrawals}
\]

or

\[
AB_{\text{new}} = AB_{\text{old}} + D - W
\]

This equation shows simply that the new account balance can be calculated by adding the deposits and subtracting the withdrawals from the old account balance. In the simplest model, \(D\) and \(W\) are constants and we end up with a linear model. However, we want to take into account that the account accrues interest, so we need to modify \(D\). Interest usually accrues as a percentage of the account balance, so the deposits due to interest (\(D_{\text{interest}}\)) will be equal to the account balance times some interest rate (IR). If we add this to the constant deposits,

\[
D = D_{\text{constant}} + D_{\text{interest}} = D_{\text{constant}} + AB_{\text{old}} \cdot IR
\]

and the new account balance can be calculated by.

\[
AB_{\text{new}} = AB_{\text{old}} + (D_{\text{constant}} + AB_{\text{old}} \cdot IR) - W_{\text{constant}}
\]
Thus, we can do a good job of describing this bank account model with a hierarchical diagram and an equation.

\[ AB_{new} = AB_{old} + (D_{constant} + AB_{old} \cdot IR) - W_{constant} \]

Figure E-8: A hierarchical diagram and iterative equation do a good job of describing our simple model for a bank account.

In the next activity, you will generate a hierarchical diagram and an equation to describe each of the models you have generated so far in this unit.

**Activity 2.3.4 Describing Models in Different Ways**

a) Generate a hierarchical diagram and an equation to describe the linear model of population growth you developed in Activity 2.2.2. Draw your diagram and write your equation below.
b) Generate a hierarchical diagram and equation to describe the exponential model for population growth that you developed in Activity 2.2.3. Draw your diagram and write your equation below.

c) Look at your equations from parts a) and b). How are they different? What about the equation in part a) makes it linear? What about the equation in part b) makes it exponential?

The equations for the linear and exponential models you have developed so far, take on the following form.

\[ P_{\text{new}} = P_{\text{old}} + \text{Births} - \text{Deaths} \]

or

\[ P_{\text{new}} = P_{\text{old}} + B - D \]

The difference between these two models is in how you go about calculating the number of births (B) and the number of deaths (D). In the linear model, B and D are fixed numbers, whereas in the exponential model, B and D are percentages of the total population. In the exponential case, B and D are calculated as follows.

\[ B = BR \cdot P_{\text{old}} \quad \text{and} \quad D = DR \cdot P_{\text{old}} \]
where \( BR \) is the birth rate and \( DR \) is the death rate. When written in this way, the birth rate and death rate are expressed in “number of births (or deaths) per person per year (or generation)”.

Throughout the rest of this unit, we will build a few more models. These models will be somewhat more sophisticated than the previous models but will be based on the same simple principles. As you work through the rest of the unit, keep in mind the basic features of linear and exponential models. In addition, when you begin to develop new models, you will likely find that the hierarchical diagrams and simple equations described in the last few pages are useful tools.

More Feedback: Incorporating the Affects of Resource Scarcity

Recall that in Activity 2.3.3 we ran into a problem incorporating the affects of resource consumption into our model. In the model we developed, population growth was unaffected by resources until the population was greater than the available resources at which time, either the population died, or it immediately leveled off to a value such that all the resources were consumed. While this kind of model reflects the fact that the population will be limited by the available resources, it does so in a somewhat unrealistic way.

Activity 2.3.5 What would happen in a real population as resources became more and more scarce?

a) Imagine an island that only had food resources to feed 200 people. What do you think would happen if the population of 300 people were shipwrecked on this island.

b) Which of the factors in your model do you think would be most affected by the scarcity of food?

c) If the population was small compared to the available resources, for example, a population of 20 on and island with food resources for a population of 200, do you think the death rate would be high or low? Do you think the death rate would be zero? Why?
d) If the population were large compared to the available resources, for example, a population of 300 on an island with food resources for 200 people, do you think the death rate would be high or low? Why?

It seems obvious that as resources become more and more scarce, the death rate should rise due to starvation and that when resources are abundant, the death rate should be relatively low, but not zero, since some people will always die of old age. Thus, we need to incorporate feedback into our model so that when there are many more resources than people, the death rate is low and when there are more people than resources, the death rate is high.

One of the easiest ways to achieve this is to make the death rate proportional to the number of people divided by the available resources.

\[
\text{Death Rate}_{\text{starvation}} = \frac{\text{Population}}{\text{Resources}} \quad \text{or} \quad DR_{\text{starvation}} = \frac{P_{\text{old}}}{R}
\]

Looking carefully at this equation, the death rate will be nearly zero when there are many more resources than people and it will be very large when there are many more people than resources. While this model reflects the variation in the death rate due to scarcity of resources, it does not take into account death due to natural causes. In order to take this into account, we should add to this death rate a constant natural death rate.

\[
DR = DR_{\text{natural}} + DR_{\text{starvation}} = DR_{\text{natural}} + \frac{P_{\text{old}}}{R}
\]

In the next activity, we will begin to incorporate this variable death rate. Remember, now that the death rate depends on the population and available resources, it will need it’s own column.

**Activity 2.3.6 Feedback: Letting the Death Rate Depend on Available Resources**

a) As in Activity 2.3.4, generate a hierarchical diagram and an equation to describe a model of population growth with a death rate that depends on the population and resources as shown in the previous paragraphs. Draw your diagram and write your equation in the space that follows.
b) Modify your spreadsheet so that it incorporates a variable death rate as described above. Begin by setting our constant death rate to something about half the size of your birth rate. Also, set up your model so that it only incorporates renewable resources. Graph your population as a function of time and describe what eventually happens to the population.

c) Describe in words what you think is happening to the population and how the resource limitation is affecting population growth.

In the last activity, you may have noticed that the population eventually leveled off to a constant value. Looking back over all of our previous models, this feature is clearly something new. If you look closely at your population vs. time graph from this model,
you will see that the population initially grows exponentially, but then eventually levels off to a constant value. This seems to make sense, since a real population cannot grow forever in an environment with limited resources.

In the next activity, we will explore this interesting feature of our new model.

**Activity 2.3.7 How do Changes in Resources Affect Constant Populations**

a) Imagine that the island population from Activity 2.3.1 found that there were enough coconuts growing on the island to feed 300 people. That is, they had 300 units of renewable food resources available. Assume you begin with 2 people, that the birth rate is 2 people per person per generation and that the initial death rate is 1.2 people per person per generation. Use the model from Activity 2.3.6, to see how this population will grow. Graph the population as a function of time, print your graph and include it in your activity guide.

b) What eventually happened to the population of islanders?

c) Imagine that after 30 generations, another shipwreck occurred near the island and 50 new people swam ashore. Add these people to the island population and observe what happens to the population. What do you observe?
d) What do you think happened to the death rate when the new people arrived? Explain what happened and what made the population returned to it’s previous value.

e) Now, imagine that 10 generations later, the population was affected by an epidemic that killed 100 people. Incorporate this into your spreadsheet and observe how the population is affected. Describe what happened to the population.

f) Imagine that after 50 generations, the population found out how to get to another small island on which they found another 200 more units renewable food resources. Modify your spreadsheet so that after 50 generations, the available food resources rises from 300 to 500. Graph the population vs. generation and include a printout in your activity guide. Describe what happened to the population after they learned to farm.

g) Now, imagine that 25 generations later, the island was hit by a bad hurricane. While everyone survived the hurricane, many of the coconut trees on both islands were blown down. Adjust your model so that after a total of 75 generations, the total available food resources drops to 350. Graph the population vs. time and include a printout here. Describe what happened to the population after the available food resources was reduced because of the hurricane.
h) Discuss with your group how you think the amount of available resources affects the value at which the population will reach equilibrium. Describe what you think is happening when the population reaches equilibrium.

Carrying Capacity

In the last two activities, you looked at a simple model that incorporated a variable death rate to account for the scarcity of resources. You saw that eventually, the population leveled off at some value that depended on the available resources. This value is often referred to as the carrying capacity. One might naively conclude that since the population has leveled off to a constant value, no one is being born or dying. However, this doesn’t make sense, since people will certainly continue to reproduce and die. Thus, when the population has reached the carrying capacity it is in a kind of dynamic equilibrium in which the number of people being born is equal to the number of people dying.

As you observed, if for any reason, the population grows beyond the carrying capacity, the death rate rises and the population decreases back, down to the carrying capacity. If, on the other hand, the population decreases, the death rate drops and the population rises back up to the carrying capacity. In addition you saw that if the amount of renewable resources is increased, the carrying capacity will rise and if the amount of renewable resources is decreased, the carrying capacity will fall.

Even though this model is fairly simple, it does a good job of reflecting how the population will respond to a variety of different changes. Thus far, you have only begun to investigate the behavior of this model. While it seems simple enough at first, it can demonstrate some surprisingly sophisticated behavior. Your instructor may show demonstrate some of the more peculiar behaviors of this simple model for your class. You may find it interesting to explore this model or an extended version of this model in more detail during the project portion of this course.
In this section, you will begin to look at global warming from a scientific perspective. Global warming is often discussed in the media and in political circles, but these discussions tend to focus on swaying public opinion through media friendly sound bites rather than developing a clear scientific understanding of the issues that surround global warming.

This section is dedicated to helping you develop a scientific understanding of global warming. The purpose of this section is to help you become more informed about the scientific basis underlying different claims about global warming. As you work through the rest of this unit, be wary of taking facts for granted. Reason through each activity and pay close attention to the details. Only by carefully investigating the facts will you be able to make an informed decision about the issues surrounding global warming.

You will need some of the following equipment for the activities in this section:

- Computer-based laboratory system (w/ computer, interface, data collection software and electronic sensors) [1.2,1.3]
- 1 IR LED source (RadioShack® 276-143c) [1.2.1]
- 1 D-cell battery, 1.4v with a battery holder [1.2.1]
- 2 alligator clip leads, approx. 4” long [1.2.1]
- 1 switch [1.2.1]
- 1 IR sensor (PASCO infrared sensor or Vernier TI Light sensor) [1.2.1]
- 1 clear rectangular watertight container [1.2.1]
- CO₂ gas tank (with rubber tube on end) [1.2.1]
- A clear plastic bottle with a lid [1.2.1]
- Desk lamp w/ a 60W or 75W incandescent bulb [1.2.1]
- 2 plastic bins, 8”L X 8”W X 3.5”D (e.g. 10 cup Rubbermaid bins w/ 11/16th” holes centered 1.5” from bottom) [1.2.2, 1.2.3]
- gravel (approx. 0.5 lb, aquarium gravel works well) and tap water [1.2.2, 1.2.3]
- 1 desk lamp w/ a 60W or 75W incandescent bulb [1.2.2, 1.2.3]
- 2 #2 stoppers w/ no hole (to seal the chamber when sensors are not present) [1.2.2, 1.2.3]
- 1 electronic temperature sensor [1.2.2, 1.2.3]
- Optional: 1 PASCO electronic IR sensor (requires PASCO interface) [1.2.2, 1.2.3]

## 3.1 WHAT IS GLOBAL WARMING?

Global warming as a topic of discussion enters our lives through many different avenues. We’ve all heard different things about global warming, what causes it, and how it will affect us. In the next activity, you will discuss your ideas about global warming with your group.

### Activity 3.1.1 What is Global Warming?

a) Discuss global warming with your group. Talk about what it is, what you know about it and why anyone cares about it. Record the main points of your discussion below and be prepared to contribute your ideas to a class wide discussion.
In addition, your group probably came up with a wide variety of ideas for what global warming is all about. In general when people talk about global warming their conversations focus on things like changes in the earth’s weather, unusually wet or dry years, el niño, hotter summers, milder winters, rising sea levels, etc. In addition, most of the discussion centers around how the weather will change over the long term, e.g. 30 years in the future. These long term changes in the Earth’s overall weather patterns fall under the classification, *global climate change*.

Scientists who study global climate change look at how the earth’s overall weather has changed in the past. Based on their knowledge of global climate patterns from the past and their understanding of the basic phenomena the drive global weather patterns, they construct models to try to predict how our climate will change in the future. The rest of this unit is designed to give you experience with a scaled down version of this kind of investigation. Throughout the rest of this unit, you will: look at how the Earth’s temperature has changed in the past, explore some of the fundamental physical principles that affect the Earth’s temperature, and develop a simple model for how the Earth’s temperature could be affected by other changes in the global environment.

*Exploring the Global Climate Record*

Scientists have compiled an enormous amount of data about the past climate of the Earth. They have drawn data from geological and archaeological sources as well as historical records of the weather. Some of the most useful sources of climatic data have included ice cores, lakebed sediment cores, and tree rings. From these and other sources, scientists have obtained a historical record of a wide variety of global climatic data from annual temperatures and precipitation amounts to the atmospheric concentrations of trace gases. In the next few activities, we will look at one component of this vast climatic record. We will look at how the Earth’s temperature has changed in the distant and recent past.
**Figure E-9:** Global temperature data for the last 400,000 years. Temperatures are reported as differences from the current average global temperature (~15 °C). Notice that the average global temperature has fluctuated dramatically over the last 400,000 years. Also note that last ~10,000 years has been characterized by a relatively high, relatively stable global temperature. This temperature data was reconstructed from deuterium isotope abundances from the Vostok Ice Core. Petit, J.R., et al., 2001, Vostok Ice Core Data for 420,000 Years, IGBP PAGES/World Data Center for Paleoclimatology Data Contribution Series #2001-076. NOAA/NGDC Paleoclimatology Program, Boulder CO, USA.

**Activity 3.1.2 Global Temperatures: Last 400,000 Years**

a) Look carefully at the graphs in Figure E-9. Describe the global temperature over the last 400,000 years. How has it changed? How much has it varied? **Note:** If you would like to get a better look at this data. It is available in the file E030102_EarthTemp.xls
b) Try to identify any significant features in the global temperature data. Do you see any periods that might correspond to ice ages? If so, during what periods?

c) Look closely at the temperature record for the last 10-12,000 years. Compare the temperatures in this recent period to the rest of the temperature record. How have the temperatures over the last 10-12,000 years differed from the rest of the 400,000 year temperature history.
d) Considering the fact that human civilization has grown up over the last 8-10,000 years, do you think that this growth of civilization was in any way influenced by the Earth’s temperature? If so, how?

In the last activity, you looked at a record of the Earth’s temperature over the last 400,000 years. You probably noticed that in much of this period, Earth’s temperature was considerably lower than it is today. In fact, the record shows that there are several periods during which the average global temperature was as much as 8 °C cooler than it is today. Geologists refer to these periods as ice ages.

You may have also noted that the Earth’s temperature has been comparatively warm and stable over about the last 10,000 years. Compared to the previous 400,000 years of dramatically fluctuating temperatures, with alternating ice ages and re-warmings, the last 10,000 years has been an incredibly stable period. The fact that that this warm stable period has coincided with the rise of human civilization may be only a coincidence, but it is reasonable to believe that a warm stable climate would be conducive to the development of agriculture that has been the foundation of our civilization.

In the last activity, we looked at how the Earth’s temperature has changed from the distant past to the relatively recent. In the next activity, we will look in more detail at how the Earth’s temperature has change in the more recent past.

Figure E-10: Global temperature data for the last 1,000 years. Temperatures are 30 year averages reported as differences from the current average global temperature (~15 °C). These data were compiled from various historical records and archaeological proxies and then spatially averaged. Jones PD, Osborn TJ and Briffa KR (2001) The evolution of climate over the last millennium. Science 292, 662-667.
Figure E-11: Global temperature data for the last 140 years. Temperatures are reported as differences from the current average global temperature (~15 °C). These temperatures were compiled from various worldwide direct observations of land and sea surface temperatures and then spatially averaged.

Activity 3.1.3 Global Temperatures: Last 1,000 Years

a) Look carefully at the graphs in Figures E-10 and E-11 of the global temperature over the last 1000 years. Describe the global temperature record for the last 1000 years. How has it changed? How much has it varied? Note: If you would like to get a better look at this data. It is available in the file E030102_EarthTemp.xls

b) Look closely at the temperature record for the last 100-150 years. Compare the temperatures in this recent period to the rest of the last 1000 years. How have the temperatures over the last 100-150 years differed from those of the last 1000 years.
In the last two activities, you looked closely at the Earth’s temperature record of the last 400,000 years. These data seem to indicate that the temperature on the surface of the Earth has varied significantly over the last 400,000 years and that the Earth has experienced a period of relatively warm stable weather over the past 10,000 years or so. The record also shows that over the last 1,000 years, the Earth’s temperature has varied but on a much smaller scale that over the past 400,000 years.

Looking closely at the last 150 years, it is clear that the Earth’s average global temperature has risen by between 0.6 and 0.8 °C. This temperature rise over the last 150 years is what scientists are referring to when they talk about global warming. But what is causing this temperature rise. Many scientists believe that this increase is due to human activity on earth, but some others believe that this increase in temperature is due to natural causes. Both groups are using this data in combination with experimental data on how the Earth’s oceans, land and atmosphere behave to get a better understanding of what might be causing this temperature rise.

Throughout the rest of this unit, we will develop a simple model to predict the temperature of the Earth’s atmosphere based on experimental data to see how scientist might begin to model global warming.

### 3.2 MODELING THE EARTH’S TEMPERATURE

Just as we did for populations, we will begin to develop our model of the Earth’s atmosphere by looking at the factors that might affect the Earth’s temperature.

#### Activity 3.2.1 What Factors Affect the Earth’s Temperature?

a) Brainstorm with your group and come up with a list of factors that might affect the temperature of the Earth. What things might cause the Earth’s temperature to rise? What things might cause the earth’s temperature to fall?
b) Group your factors based on which ones most directly affect the temperature of the earth and construct a hierarchical diagram of the factors that affect the Earth’s temperature. **Note:** Remember to construct this diagram based on what factors most directly affect the Earth’s temperature.

c) Based on your diagram, devise a very simple model that you think might describe the behavior of the Earth’s temperature. **Note:** Be prepared to discuss your model with the rest of the class.

Checkpoint Discussion: Before proceeding, discuss your ideas with your instructor!
In the last activity you developed a simple model for how the temperature of the earth changes. You probably recognized that the most significant factor influencing the temperature of the earth is the sun. The sun shines on the Earth and “heats it up.” But how does the sun heat up the Earth, and why doesn’t the Earth keep getting hotter and hotter?

In the next activity, we will work toward answering these questions using a simple model of the Earth-Sun system. We will use a standard desk lamp to represent the sun and a small tray of gravel to represent the earth.

**Activity 3.2.2 Modeling the Sun’s Affects on Earth?**

a) Turn the lamp on and hold your hand underneath it (3-6 inches away). What do you feel? Now close your eyes and have one of your group mates place a piece of paper between the lamp and your hand. What do you feel? What did the piece of paper do?

b) How, specifically, do you think the lamp causes your hand to heat up?

c) How do you think the sun causes the earth to heat up?

Earlier in this unit, you developed several kinds of population models. These models had three primary constituents:
1. A Level: The value or quantity of interest, for example, the population, amount of available interest, or bank account balance.
2. An Input: The mechanism by which the Level is increased, for example, the number of births per year, or the amount of deposits.
3. An Output: The mechanism by which the Level is decreased, for example, the number of deaths per year, the amount of resources consumed per year, or the amount of withdrawals.

Figure E-12: A generic hierarchical depicting an abstract model with an Input, a Level, and an Output.

In addition, you found that there were several different kinds of inputs and outputs.

Linear Inputs (or Linear Outputs) that added (or subtracted) a fixed amount to the level during each time interval. A graph of the level vs. time for a system with linear inputs and outputs was a straight line.

Exponential Inputs (or Exponential Outputs) that increased (or decreased) the level by a fixed percentage during each time interval. A graph of the level vs. time for a system with exponential inputs and outputs was a smooth exponential curve with an ever increasing or decreasing slope.

Variable Inputs (or Variable Outputs) that increased (or decreased) the level by a variable amount during each time interval. Because there are so many different types of variable inputs and outputs, the level vs. time graphs of systems with these kinds of elements do not have any particular shape. However, you can bet that if your system doesn’t look linear or exponential, it has either a variable input or a variable output.

Figure E-13: Equation and graphical representations of three types of inputs and outputs: linear, exponential, and variable.

Thus far, we have devised a simple model to predict how the Earth’s temperature will change. We have decided that the most important input for our model is the sun and that since things naturally cool down, this natural cooling is our output. In the next activity,
you will observe how a model sun (a desk lamp) heats up a model earth (a tray of rocks) and try to characterize which kind of input best describes the sun.

### Activity 3.2.3 Characterizing the Sun as an Input

**a)** Set up your model sun and earth. Poke a small hole through the side of a plate with a pair of scissors. Then push a temperature sensor through the hole in your plate and tape it in place so that the tip of the temperature sensor is near the center of the tray. Place a small amount of gravel (about one layer of rocks, too many rocks will make things heat up too slowly to see easily) in the tray and put the tray underneath your desk lamp so that the lamp 2-4 inches from the gravel. DO NOT TURN THE LAMP ON YET. Predict how you think the temperature of the tray of rocks will vary with time after you turn the lamp on. Sketch a rough graph of your prediction below. What kind of input source do you think the sun is?

**b)** Now, try to the experiment. Open the file E030203.mbl and begin recording. Turn on the lamp. The computer will stop recording after 200 seconds. **Note:** Leave the lamp on after the computer has stopped collecting data. Describe the graph of temperature vs. time.

**c)** Based on this data, how would you characterize the lamp as an input?
d) If the lamp does a good job of representing the sun, what kind of an input do you think you should use to model the sun?

You just characterized the sun as an input source into your model. In the next activity, you will look more closely at how the Earth might cool and characterize the natural cooling of the Earth as an output for your model.

**Activity 3.2.4 Natural Cooling as an Output**

a) Predict how you think your model Earth (tray with rocks) will cool when you turn off the lamp. Predict how you think the temperature will change as a function of time and graph a rough sketch of your prediction below. What kind of output do you think the natural cooling of the rocks will be: linear, exponential or variable? Explain your reasoning.

b) Open up the file E030204.mbl and begin collecting data yet. Turn of the lamp and move it out of the way. Place your hand over the rocks (1/2 to 1 inch away, but do not touch). Describe what you feel.
c) How do you think the rocks cause your hand to heat up?

d) If the process for natural cooling of the rocks could be best modeled by a linear output, how would it look. Draw a graph of temperature vs. time in the following space.

e) If the process for natural cooling of the rocks could be best modeled by an exponential output, how would it look. Draw a graph of temperature vs. time in the following space.
f) How did the rocks actually cool. Draw a rough sketch of the temperature vs. time below. How would you characterize the natural cooling of the rocks as an output?

g) Put your hand over the rocks (1/2 to 1 inch away, but do not touch). Compare what you feel to what you felt in part b). How has the amount of “light, heat, energy, stuff” coming off of the rocks changed as the rocks cooled?

Checkpoint Discussion: Before proceeding, discuss your ideas with your instructor!

Now that you have characterized the sun as an input and natural cooling as an output, you can put these to pieces of information into your model for Earth’s temperature. In the next activity, you will build a spreadsheet model that is analogous to your Earth Temperature model to predict how the temperature of the thermometer will change. You will then perform an experiment and try to make your model reflect the observed temperature change.

Activity 3.2.5 Testing Your Model for the Earth’s Temperature

a) Synthesize the information you’ve learned in the last few activities and devise a model for how the Earth’s temperature will change. Come up with an equation for how you will calculate the new temperature ($T_{\text{new}}$) from the previous temperature ($T_{\text{old}}$). Write your equation below.

\[
T_{\text{new}} =
\]
b) In this activity you will perform a simple experiment to test your model, but first you must build a spreadsheet model to make a prediction for how your temperature will change.

In the experiment, you will place a thermometer about 2-3 inches below the desk lamp. You will record the temperature for a total of 600 seconds. You will turn on the desk lamp for the first 300 seconds and then turn off the desk lamp for the final 300 seconds.

Based on your model for the Earth’s temperature, build a spreadsheet model to predict how the temperature of the thermometer will change over the course of this 600 second experiment. Graph the temperature as a function of time, print out your graph and include it in your activity guide.

c) Now perform the experiment. Tape your temperature sensor in place on the table so that it cannot move around, but so that the tip of the temperature probe does not touch the table. Place the lamp 2-3 inches above the sensor. Open the file E030205.mbl, and begin recording. Turn on the desk lamp. After 300 seconds, turn off the desk lamp. Print out a temperature vs. time graph for your experiment and include it in your activity guide. Note: We are using a bare temperature probe because it will respond more quickly than the temperature probe in the tray of rocks. If we had used the temperature sensor in the tray of gravel instead, we would get the same results, but it would have take about twice as long.

d) Compare your prediction with the experiment. How are they different? How are they similar?
e) Based on your experimental results, modify the parameters in your model so that the temperature vs. time graph for your model closely resembles the one you obtained from your experiment. Note: If you were very careful with your experiment should be able to get your model to fit the experimental data quite well. If you are having trouble fitting your model to the experimental data, you may need to take into account the fact that the experimental system will only cool down to room temperature. One way to solve this problem is to replace $T$ by $(T - T_{room})$ everywhere in your model.

Checkpoint Discussion: Before proceeding, discuss your ideas with your instructor!
3.3 **THE ROLE OF LIGHT ENERGY IN GLOBAL WARMING**

In the last few activities, you developed a simple model for how the Earth’s temperature is affected by the sun and natural cooling. When trying to explain how the sun causes the Earth to heat up, you probably argued that some light, heat, energy stuff comes from the sun and hits the Earth. This is a reasonable explanation that we will look at in more detail in the activities that follow.

We noticed in previous activities that when we placed our hand under a lamp, it was not only illuminated, but also warmed. When a piece of paper was placed between the lamp and our hand, our hand was shaded, but also cooled. We notice this same behavior with the sun. On a sunny day, it is very light and we feel warm, but if a cloud blocks the sun momentarily, not only does it get darker, we also feel somewhat cooler. Clearly, light comes from the sun, but the sun also makes us feel warm, how does this happen.

For the rest of this unit, we will refer to the light, heat, energy stuff coming from the sun as *light energy*. As we try to build a more complete model of global warming, it seems prudent to learn more about the nature of the light energy coming from the sun. In the next few activities, you will begin to look more closely at the nature of this stuff coming from the sun. In particular, the next activity, will look at how the light we see is related to the heat we feel from the sun.

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**Activity 3.3.1 The “Temperatures” of Different Colors of Light**

a) Place your prism in a clamp and adjust it so that the sun shines on it and it casts a rainbow on a white sheet of paper. It may take some fidgeting to see a rainbow. When you do, adjust the rainbow to get as large a rainbow as possible with the colors still very bright. Describe what is happening to the light from the sun as it strikes the prism and becomes a rainbow. Why don’t we see a rainbow when the sun shines directly on the paper, but we do see a rainbow when it shines through a prism. **Note:** Weather permitting, this activity will be done outside on a sunny day. Otherwise, your instructor will demonstrate the apparatus and provide you with data taken under more suitable weather conditions.
b) If we place three thermometers in the rainbow as shown (one in the blue, one in the red, and one in the shadow beyond the red), which do you think will record the highest temperature? Record your predictions below and explain your reasoning.

Highest Temp: ____________  
Middle Temp: ____________  
Lowest Temp: ____________

c) Now, perform the experiment. Blacken the tips of three temperature sensors so that they will absorb more light. Tape the temperature sensors to a sheet of white paper so that they are about 1-2 cm apart. Place them in a cardboard box (a copier paper box works well). Remember to keep your temperature sensors in the shade. Open the file E030301.mbl and begin recording data. After a few seconds, adjust the prism so that it casts a well defined rainbow on the temperature sensors as shown above. Make sure that one sensor is at the blue end of the spectrum, one sensor is at the red end of the spectrum and one sensor is in the shadow beyond the red end of the spectrum. Note: There are three possible ways to perform this experiment. If available, you can use your data acquisition system in remote mode so that you can take the unit outside without a computer. If your unit doesn’t support this feature, you can collect data outside with a laptop. Otherwise, this experiment can be done with three regular thermometers with blackened bulbs. Also note that blackening with paint works best, but that permanent marker can be used.
d) What do you observe? After the temperatures have settled down, measure the average temperature for each temperature probe and record them below. How does this compare with your prediction.

Blue Average Temp: ____________
Red Average Temp: ____________
Beyond Red Average Temp: ____________

e) Based on your observations, what can you conclude about the region beyond the red end of the rainbow.

This experiment was first performed by William Herschel in 1800. Based on his results, he concluded that there must be some kind of light energy coming from the sun that we cannot see. Today, we call this kind of light energy infra-red (or IR) because it can be found beyond the red part of the rainbow.

Since Herschel’s first experiment, we have developed a variety of different ways to detect IR light energy. One way to detect IR is with a common digital camera. In Figure you can see some pictures of the experiment you just performed taken with a digital camera. The first picture was taken in the usual fashion, but the second picture was taken with a special filter that only passes IR light energy and screens out all of the visible.

Figure E-14: Pictures taken of the three temperature sensors used in Activity 3.3.1. Both of these pictures were taken with a standard digital camera. The bottom picture was taken with a Hoya R72 Infrared pass filter. You can see that the bottom temperature sensor which appears to be in shadow in the picture on the left is actually illuminated by IR light energy in the picture on the left. IR light energy extends even farther beyond the
red than is shown in the above picture, however, standard digital cameras are not sensitive to this “Far IR” light energy. As a result, a temperature sensor placed even a bit farther beyond the red will still heat up even though it would appear to be in shadow in the rightmost image above.

It turns out that the light energy you observed in the last activity makes up only a small portion of the light energy that reaches us from the sun. There are many kinds of light energy including: radio waves, microwaves, infra-red light, visible light, ultraviolet light, x-rays, and gamma rays. Each of these types of light energy can be associated with a different wavelength.

Figure E-15 shows the names assigned to various wavelengths of electromagnetic spectrum from the shortest to the longest. Cosmic rays that have a tiny wavelength of about $10^{-14}$ meters (or 0.00000000000001 meters), are the shortest form of electromagnetic radiation emitted by the sun. Radio waves, with a wavelength of 10 meters or more, are the longest.

**Figure E-15:** The Electromagnetic Spectrum. All of the different types of light energy presented here are qualitatively the same. They are all made up of oscillating electric and magnetic fields. What makes them different is the wavelength of their oscillation.

### 3.4 What happens to visible and IR radiation as it passes through the Earth’s atmosphere?

The earth’s atmosphere absorbs or reflects about half of the light energy from the sun. Some gases present in the Earth’s atmosphere are very efficient at absorbing certain types of light energy. Just because a particular substance absorbs one type of light energy doesn’t mean it necessarily will absorb another type. Some gases in the atmosphere transmit visible light, but absorb infrared. These gases are called greenhouse gases. Two notable greenhouse gases are water (H$_2$O) and carbon dioxide (CO$_2$). In the next activity, we will investigate some of the absorption characteristics of water.

The purpose of this experiment is to measure how well water transmits visible and infrared light. You should set up your infrared light source (LED) as shown in Figure E-16. Setup the infrared LED, the infrared sensor and plastic container as shown in Figure E-17. Set up the sensor and LED so that two plastic containers can fit between...
them. Adjust the position of the sensor so that signal is a maximum and tape the LED and sensor down so that they do not move when you empty the water to the plastic container.

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**Activity 3.4.1 Can a Material Transmit Visible Light and Block IR?**

a) Fill a beaker with water and look at it carefully (using your eyes as a sensor). Is visible light transmitted through the glass in the beaker? Is visible light transmitted through the water? How can you tell whether or not visible light passes through the materials?

b) Now, let’s measure whether or not water blocks IR radiation. Empty the beaker and switch the battery to the on position so that the LED is emitting infrared radiation. Since you cannot see IR radiation, you will use an electronic sensor. Open the MBL file A020401 and record the IR sensor reading. **Note:** Try to line up the IR source and sensor so that the maximum amount of radiation gets to the sensor. To do this, watch the output of the IR sensor on the screen and adjust the position of the source until the reading reaches a maximum value.

c) Predict what you think will happen to the sensor reading as you add water. Explain the reasoning behind your prediction.

d) Now add a small amount of water to the beaker (about 50 ml). What happens to the sensor reading?
e) Based on the observations you made in the first two parts of this activity, what can you say about how water interacts with visible and infrared radiation. Is it possible for the molecules in a substance to transmit visible light and at the same time absorb infrared light?

f) Do you think its possible for visible radiation from the Sun to pass through the Earth’s atmosphere at the same time that the Sun’s infrared radiation is being absorbed? Explain.

Checkpoint Discussion: Before proceeding, discuss your results and conclusions with your instructor.

In the last activity you saw that water is transparent to visible light, but is very efficient at blocking infrared radiation. In this sense, the water in the last activity is very much like a greenhouse gas. But the analogy between this experiment and the atmosphere is limited because we are comparing a situation without greenhouse gases to one with greenhouse gases. But what will happen if the amount of water between the LED and the

Activity 3.4.2 What happens if the amount of absorber is increased?

a) Predict what you think will happen to the sensor reading if you double the amount of water between the sensor and the IR LED?
b) Record the sensor reading from the last activity in the table below. Add another 50 ml of water and record the new sensor reading. Repeat this process two more times until you have 200 ml of water in the beaker.

<table>
<thead>
<tr>
<th></th>
<th>IR Sensor Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty Beaker</td>
<td></td>
</tr>
<tr>
<td>50 ml H₂O</td>
<td></td>
</tr>
<tr>
<td>100 ml H₂O</td>
<td></td>
</tr>
<tr>
<td>150 ml H₂O</td>
<td></td>
</tr>
<tr>
<td>200 ml H₂O</td>
<td></td>
</tr>
</tbody>
</table>

c) What does this suggest will happen if we increase the amount of absorbing greenhouse gases in the environment? Explain your reasoning.
A Schematic Diagram of Light Energy in the Earth’s Atmosphere

To study the effect of human activity on global climate change properly, would require more time and sophisticated equipment than we have available for this class. Still, based on the observations we have made, we can construct a simple scientific model of how light energy interacts in the Earth’s atmosphere and how this might lead to global climate change. One way to construct this model is to track what happens to a single chunk of light energy that comes from the sun and strikes the Earth.

To simplify our model, we will begin by imagining a dark earth with no sun. In this situation, the Earth would begin at a very low temperature. Now, imagine that the sun turns on. The sun begins to emit chunks of light energy that contain all colors of light including visible and infrared radiation. What will happen to a chunk of light energy that leaves the sun and hits the Earth.

1. Light energy including visible and infrared light energy comes from the sun and enters the Earth’s atmosphere.
2. Some of this light energy is reflected by clouds or absorbed by gases in the upper atmosphere, while some of the light energy passes through the Earth’s atmosphere.
3. The light energy (both visible and IR) that is not reflected or absorbed by the atmosphere is absorbed by the earth. This energy is then converted into thermal energy and causes the temperature of the earth to rise to a somewhat higher temperature.
4. But since the temperature of the earth has risen it emits some IR light energy.
5. This emitted IR radiation travels up through the atmosphere. Some of it escapes and is radiated away from the earth, but some is absorbed by greenhouse gases in the atmosphere and reradiated.

The net effect of this one chunk of light energy hitting the earth is to raise the temperature of the earth slightly converting some of the light energy to thermal energy which is stored in the earth. This stored thermal energy is evidenced by the fact that the temperature of the Earth has risen slightly. The remainder of the light energy that hit the earth has been converted to IR light energy that is then emitted from the earth into space.

Now, imagine that many chunks of light energy come from the sun and strike the earth. As more and more chunks strike the Earth, the temperature will rise because the amount of energy coming to the Earth from the sun each second is greater than the amount of IR emitted by the Earth each second due to its temperature. But why doesn’t the temperature continue to rise forever. In order to understand this fully, we need to take into account the fact that as an object gets hotter, it emits more IR light energy and as it cools, it emits less light energy.

In fact, you did just this in the models you developed in Activity 3.2.5. You’ll recall that in this activity you used the following model.

\[ T_{\text{new}} = T_{\text{old}} + \text{IN} - \text{OUT} = T_{\text{old}} + \text{IN} - (\text{OutR} \cdot T_{\text{old}}) \]
In this model, your cooling rate decreased as the temperature of the Earth decreased and increased when the temperature of the earth increased. This behavior mimics the IR output of the Earth very closely. And in fact, it should since one of the primary ways that objects cool (lose thermal energy) is by emitting IR light energy.

Since the cooling rate increases as the temperature increases, the cooling (OUT) rate will eventually equal the rate of light energy input from the sun (IN). When this happens, the amount of energy coming to the Earth from the sun is equal to the amount emitted from the Earth because it is warm. At this point, the temperature levels off because the amount of energy coming in is equal to the amount going out.

The Greenhouse Effect

As you learned in Activity 3.4.1 and Activity 3.4.2, some materials can transmit visible light while still absorbing infrared radiation. Gases that have this property are called greenhouse gases. The atmosphere contains a variety of greenhouse gases including water vapor, methane and carbon dioxide. While water vapor is, by far, the most important greenhouse gas, there is little we as humans do to change the amount of water vapor in the atmosphere as a whole. On the other hand, it is fairly clear that the human population has dramatically changed the concentration of CO₂ in the atmosphere by burning fossil fuels, since one of the major by products of fossil fuel combustion is CO₂.

The data in Figure E-18 clearly shows that we have dramatically increased the amount of CO₂ in the atmosphere over the past 40 years. But how will this affect the Earth’s climate?

![Figure E-18: Monthly CO₂ levels as measured at Mauna Loa Observatory in Hawaii. This data has been taken continuously since 1959.](image)

We know that since greenhouse gases absorb IR light energy, the rate at which IR light energy will be able to leave the earth will be decreased.

In the next activity, you will work with your instructor to construct a model for what will happen to the temperature of the Earth if the CO₂ concentration in the atmosphere increases.
Activity 3.4.3 Modeling the Greenhouse Effect and Global Warming

a) In this activity, you will work with your instructor to construct a model for how increased CO₂ levels in the atmosphere might affect the earth’s temperature. The models you develop will be based on those you generated in Activity 3.2.5. As you work through these models, keep good notes on what you observe, how you expect the models to behave and how they actually behave.

Describe each of the models, record your predictions when appropriate, write down a diagram and an equation for each model and take note of any important features of each model.

In Only:

Equation:

Diagram:

Prediction:
Actual Behavior:

![Blank Graph](image)

Notes:

In and Out:

Equation:

Diagram:
In and Out + CO₂:

Equation:

Diagram:

Prediction:
Actual Behavior:

Behavior with CO$_2$ Data:

Notes:
After looking at these models, it may seem like greenhouse gases are all bad. However, this would be a rash conclusion. While an overabundance of greenhouse gases will likely result in an overall increase in global temperatures, a scarcity of greenhouse gases can lead to lower overall global temperatures. As the diagram below shows, without any greenhouse gases, the average surface temperature of the earth would be a chilly -18°C.

![Diagram](image)

**Figure E-19**: A simplified diagram of elements of the greenhouse effect. The graph on the left depicts what would happen if there were no greenhouse gases such as carbon dioxide or water vapor in the atmosphere. The average temperature at the Earth’s surface would be too cold at –18°C. The diagram on the right shows how the current level of greenhouse gases serves to maintain an average temperature of +15°C. As more carbon dioxide builds up in the atmosphere, the surface temperature is expected to rise.

We’ve looked at a lot of information throughout this unit and developed a number of simple but useful models to help us get a better understanding of how populations grow and how the Earth’s temperature can change. While no one has come up with a perfectly accurate model for either system, we have learned a lot about how these systems behave by looking at these simple models. The principles we have learned over the course of this unit are not restricted to global population growth and climate change, but are rather general. They can and have been applied to a variety of problems from high energy and nuclear physics to economics. With any luck, the general modeling methods you have learned here will serve you well when you need to understand another system who’s underlying dynamics are similar to those of the systems we have investigated here.