

THE NUTS AND BOLTS OF WRITING MATHEMATICS

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Mathematics must be written so that it is impossible to misunderstand, not merely so that it is possible to understand.

—Herman Rubin

You know that I write slowly. This is chiefly because I am never satisfied until I have said as much as possible in a few words, and writing briefly takes far more time than writing at length.

—C. F. Gauss

I would not have made this so long except that I do not have the leisure to make it shorter.

—B. Pascal

Why does Milnor get to prove all the easy theorems?

—Anonymous comment about John Milnor who is known for his short, elegant, and deep proofs.

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- (1) **Your ultimate goal.** Your aim is to succinctly and convincingly communicate a mathematical argument to the reader.
 - (2) **This is writing.** When writing mathematics you must obey all of the usual writing rules—spelling, grammar, punctuation, sentence structure, etc.
 - (3) **Not a creative writing exercise.** Avoid flowery, imprecise, descriptive, and vague language.
 - (4) **Know your audience.** The way you write a proof should depend on who will read it. In a mathematics course you should write so that your classmates can understand your proof—it should not be written for the professor or for high school students.

- (5) **Find your voice.** Mathematics is always written in first person plural (we, us, our), not first person singular (I, me, my) or third person singular (one). Unlike most sciences, mathematicians do not use the passive voice. Mathematics is usually written in the present tense.

Yes: "Using the quadratic formula we find the root of $p(x)$ to be 2."

No: "Using the quadratic formula one finds the root of $p(x)$ to be 2."

No: "Using the quadratic formula I found the root of $p(x)$ to be 2."

No: "Using the quadratic formula the root of $p(x)$ was found to be 2."

- (6) **Symbols vs. words.** You should read the mathematical symbols as words. For example, in the following sentence, the two " $>$ " symbols are verbs.

Since $2x > 4$, it follows that $x > 2$.

However, symbols should not replace words in a sentence.

If all three angles of a triangle are $=$, then it is equilateral.

You may want to read your proof aloud (including the equations)—this is a great way to find sentence fragments.

- (7) **Too many symbols/too many words.** It takes some practice to determine when to write the mathematics in words, and when to write in symbols.

Too many words: The integral of $2x + 3$ from zero to one is four.

Too many symbols: We say that $\lim_{x \rightarrow a} f(x) = L$ if $\forall \varepsilon > 0, \exists \delta > 0$ s.t. $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.

- (8) **The devil is in the details.** If the reader is mathematically sophisticated, you can often omit certain details, especially straightforward calculations. The work on your scrap paper does not always have to appear in your proof.

Too many details: Suppose $x^2 - 2x - 3 = 0$. Factoring the polynomial we obtain $(x - 3)(x + 1) = 0$. Setting both sides equal to zero we have $x - 3 = 0$ and $x + 1 = 0$. Solving both equations we find that $x = 3$ or $x = -1$.

Better: Suppose $x^2 - 2x - 3 = 0$. By factoring the polynomial and setting both terms equal to zero we find that $x = 3$ or $x = -1$.

Another example that is appropriate for a post-calculus course: The derivative of $f(x) = x^2e^x$ is $f'(x) = (x^2 + 2x)e^x$.

You should always include the details for any technique that you have just learned in your class.

- (9) **Can I have an example of that?** No. Do not put examples in the middle of your proof to illustrate your argument. The proof should consist only of your well-crafted logical argument.
- (10) **Let it flow.** Use the following words and phrases to help your writing flow: therefore, thus, so, hence, consequently, accordingly, it follows that, we see that, from this we obtain, moreover, provided that, notice that, note that, recall, since, because. Special tip: if you want to use the word “get,” use the word “obtain” instead. It always sounds better.
- (11) **State your intentions.** If you are about to make a multi-sentence mini-argument within your proof, you may want to announce your plan at the start.

Our first objective is to show that no maximum occurs in the region...

- (12) **$\sin(x)$ should not start a sentence.** Do not start a sentence with a mathematical expression or a number.
- (13) **Be concise.** A longer proof does not guarantee clearer writing. Often short, clear arguments are better than long, winding descriptions.
- (14) **Reread what you have written.** Always do this! (And, as much as it may pain you, rewrite it if necessary.)
- (15) **Practice, practice, practice.** As with any worthwhile endeavor, you must practice in order to improve.